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# THE MATHMATE

*The Official Journal of the*
*South Carolina Council of Teachers of Mathematics*

## Special Agents of ΔAcademy

# THE MATHMATE

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**2021 SCCTM Fall Conference**
**Special Agents of Δ Academy**

**November 11 - 12, 2021**
**Columbia Metropolitan Convention Center**
**in Columbia, South Carolina**

# THE MATHMATE

**Mission Statement:** The mission of The MathMate is to feature articles about innovative mathematical classroom practices, important and timely educational issues, pedagogical methods, theoretical findings, significant mathematical ideas, and hands-on classroom activities and make this information accessible to students, educators, and administrators.

**Submission Requirements**: All submission are to be emailed to mathmate@scctmconference.org as attachments along with a completed Submission Coversheet. Submitted files must be saved as MSWord or PDF files. Pictures and diagrams must be saved as separate files and appropriately labeled. Authors are asked to not submit the same article to another publication while it is under review for The MathMate.

**Submission Deadlines**: The MathMate is published three times per year. Submissions received by November 1 will be considered for the January issue, March 1 for the May issue, and July 1 for the September issue.

**South Carolina Recertification Credit**: According to the SC Department of Education Renewal Credit matrix, the primary author of a peer reviewed journal article can earn 60 renewal credits.

**Subscriptions:** Active members of SCCTM receive online subscriptions to The MathMate as part of their membership.

**Correspondence:** All correspondence regarding The MathMate should be sent to mathmate@scctmconference.org.



**About the Cover:** Graphic found in Google image search for “teaching math online”. www.google.com

# THE MATHMATE

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# MESSAGE FROM SCCTM PRESIDENT

Dear friends in mathematics,

It is with a heart full of gratitude that I write this letter today! I am thankful for…
…our Executive Board - hardworking and full of a never ending supply of amazing ideas.
…our current and previous members for sticking with us.
…the opportunity to serve the membership and others who take part in the activities we offer.
…the four Keynote speakers who have agreed to share their time and talent during the SCCTM Annual Conference being held in person November 11th and 12th this year!
…Leigh Martin for agreeing to serve as the newest editor of our SCCTM journal and for her willingness to help us transition to a hybrid newsletter/peer reviewed journal crossover.
…and, last but certainly not least, for Marc Drews for bringing together a series of conversations and presentations in place of our traditional conference during these COVID days.

The team of educators, coordinators, coaches, and leaders elected to your Executive Board are working on some amazing options for participation, opportunities for professional development, submitting grants, and making connections to support each current and potential member of SCCTM in so many ways.

We hope you will ALL be able to join us face-to-face in November to share your experiences and talents as well! Our annual conference theme this year, **Special Agents of Δ Academy**, is certain to bring you a plethora of classroom ideas, support for your own learning, and, of course, tons of fun! Be sure to submit a proposal to share your talent with us! Alternatively, if you'd like to attend but not speak, be sure to take advantage of our Early Bird Special Registration prices!

Don't want to present at the conference but would like to contribute your time or talent? Perhaps you'd like to consider running for an office! We will be sending information in a future eBlast in the coming months with the list of offices and where to send your nomination of others or to self-nominate!

With highest regards and much gratitude,

**Ryan Higgins**, President

# MESSAGE FROM SCCTM PRESIDENT ELECT

Dear Members,

As we near the end of a unique and challenging year in education, I hope that you will take the time to reflect on all that you have accomplished. I am impressed and motivated daily by the hard work and dedication of my colleagues across the state. From monitoring and implementing all covid protocols to changing the entire structure of the educational platform, you all did it with very little help. Educators are the epitome of resilience.

It is our goal at SCCTM to help support teachers and promote mathematics education by offering a researched-based, professional publication. The board of SCCTM would like to thank all authors for sharing their knowledge by writing for The MathMate. We would also like to thank Leigh Martin for serving as the editor for this edition. We hope you will consider contributing to this publication by sharing engaging lessons, interesting activities, or classroom strategies. Your work can benefit other mathematics educators in our state.

Our annual conference will be in Columbia, November 11-12, 2021. After the past year, I look forward to this face to face conference. I hope that you will take advantage of the discounts and register early. Visit our website for more information: http://www.scctm.org/.

Sincerely,

**Alisa Hobgood**, President Elect

# Message from the Editor

This issue of *The MathMate* incorporates practitioner articles, including lesson plans and activities, with peer-reviewed research-based articles. The South Carolina Council of Teachers of Mathematics is excited to transition *The MathMate* to a hybrid journal that includes both types of articles.

While all submissions in this issue were peer reviewed, Supporting Productive Struggle in Middle Grades Mathematics (pages 10-18) underwent a peer review process in line with the peer review process of other professional journals. Moving forward, we will offer authors a choice of how they would like their submissions to be reviewed. We understand that some submissions, especially lesson plans and activities, will not need to undergo the same review process as research-based manuscripts. So, if you have lesson ideas and have been hesitant to submit those ideas in the past, now is the time to consider sharing your ideas and activities with your peers in *The MathMate*!

The SCCTM Board encourages all members to consider submitting an article, lesson plan, activity, or lesson strategies to be included in the September issue of *The MathMate*. In addition to receiving recertification credit, this is a wonderful way to share ideas with peers and learn from others. We hope you will consider submitting your ideas to be included. All submissions should be shared via email, mathmate@scctmconfrence.org, by July 1.

Thank you,

**Leigh Martin**, Editor of *The MathMate*

# ANNOUNCEMENTS

Award Nomination Deadlines:

**Outstanding Contribution to Mathematics Education Award**
Nomination deadline: July 15
scctm.org/Awards

**Richard W. Riley Award**
Nomination deadline: July 15
scctm.org/Awards

Scholarship Deadlines:

**Preservice Scholarship**
Applications deadline: September 15
scctm.org/scholarships

**Educator's Scholarship**
Application deadline: September 15
scctm.org/scholarships

Membership News:

Renew your NCTM membership online and designate *South Carolina Council of Teachers of Mathematics* for the affiliate rebate.

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# 2021 SCCTM Fall Conference: Keynote Speakers

**2021 SCCTM Fall Conference**
**Special Agents of Δ Academy**

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**Dr. George Roy** is an Associate Professor of Mathematics Education at the University of South Carolina. He received his undergraduate degree in Mathematics from Rollins College, his Masters of Education in Mathematics Education from Lockheed Martin/University of Central Florida (UCF) Academy for Mathematics and Science. He went on to receive his PhD from UCF where his doctoral research focused on preservice teachers' development of mathematical content knowledge. Dr. Roy is a Keynote speaker during this year's annual conference because of his role as an author on the NCTM publication, *Catalyzing Change in Middle School Mathematics: Initiating Critical Conversations*.

**Dr. Brandie Waid** is an independent math coach/consultant who uses her life experiences in her work with districts, schools, universities and educators to support the use of teaching practices that honor each student as an individual. She was featured during a NCTM February 2021 webinar discussing identity in the mathematics classroom which is "…largely influenced by queer pedagogy, methods of re-humanizing mathematics, and essential teacher competencies…" (visit her website at www.thequeermathematicsteacher.com for more information). Dr. Waid's focus on identity in an era of change is exactly the emphasis we need for our theme this year.

**Glenna Tabor** is an innovative mathematics educator and motivational speaker. She has been a teacher, leader, and consultant and has developed a classroom framework for small-group mathematics teaching. As her website states, "Glenna sees her mission as being an agent of positive change" (visit her website at https://glennatabor.com/ for more information). Since our SCCTM conference theme centers on the idea of change, Glenna is a natural fit.

**Dr. Margaret Gilmore** is the Superintendent of Allendale County School District. Originally serving as Allendale County's transformation coach, placed by the state in 2017, Dr. Gilmore assumed the role of Interim Superintendent in 2018. She has a degree in elementary and special education as well as a Master's in Education from Arkansas State University. Dr. Gilmore received her doctorate in educational leadership and policy from the University of Memphis. Dr. Gilmore was recently identified by the SC Athletic Administrators Association as Superintendent of the Year and has been said to "really fire up" those she speaks to and leads. We are excited that Dr. Gilmore has agreed to be our final keynote speaker for the 2021 annual conference.

If you would like your announcement to appear in the next issue of The MathMate, please email all information to mathmate@scctmconference.org. Announcements will be published at the discretion of The MathMate Editorial Board.

# Supporting Productive Struggle in Middle Grades Mathematics

by Bridget Coleman, Ph.D., Nancy Ruppert, Ph.D., and Denise Johnson, Ed.D

**Abstract**

Early adolescents need opportunities for productive struggle to develop their problem solving skills. This article shares ways teachers can provide intentional experiences and opportunities for students to describe, discuss, and justify their thinking while working through math problems.

Productive struggle should be by definition, "productive" - stretching students' thinking and performance with a degree of challenge. ST Math (2021) defines productive struggle as "the process of effortful learning that develops grit and creative problem solving." According to the National Council of Teachers of Mathematics, productive struggle is one of eight mathematical research-based teaching practices (Steele, 2020) and it is noted in the Standards for Mathematical Practices in the Common Core State Standards for Mathematics. In NCTM's (2014) *Principles to Action*, a description of productive beliefs is shared: "An effective teacher provides students with appropriate challenge, encourages perseverance in solving problems, and supports productive struggle in learning mathematics" (n.p). Regardless of the level of academic prowess or experience, all young adolescents should have opportunities to be challenged and receive support and encouragement. Too often productive struggle has been the gift of more advanced students. All students benefit from productive struggle (Blackburn, 2018).

Productive struggle is a form of active learning; it is a process where students are presented material or a question or a scenario and they systematically work as an individual or with teammates to investigate problems. According to Steele (2020), productive struggle allows students opportunities to connect thinking with understanding, enhance perseverance with structures of time, and is used to develop confidence. This article will explore the structures of two math problems and how they develop students' persistence and resilience in pursuing and attaining the learning goal or understanding.

**Teachers Use of Productive Struggle to Build Mathematical Knowledge**

Teachers who promote productive struggle in the classroom embrace standards and essential attributes. NCTM's five fundamental process standards - communication, reasoning and proof, problem solving, representation, and connections - guide the type of experiences we provide in mathematics classrooms. Communication as a process standard is key to promoting productive struggle. As an extension of Vygotsky's Social Learning Theory, Driver, Asoko, Leach, and Scott (1994) found that students' knowledge is constructed through discourse, activity, and support by others. Teachers who utilize this approach provide opportunities for students to wrestle through problems alone and together as well as structure activities that help the learner conceptualize key facts and concepts. Both small group and whole class interactions play an important role in this active process.

The South Carolina Mathematical Process Standards (2015) describe specific ways in which students should individually and collaboratively engage with productive struggle. These process standards rely on meaningful problem solving experiences and require students to go beyond echoing what the teacher says. Additionally, the Association for Middle Level Education (AMLE) acknowledges the importance of classrooms that are engaging, empowering, and

challenging for young adolescents to succeed. Bishop and Harrison (2020) provide a description of essential attributes that support successful middle schools, and explain that classrooms must be developmentally appropriate, challenging, empowering, equitable, and engaging. Kang (n.d.) describes eight teaching habits that address productive struggle. A fundamental characteristic of classrooms that embrace productive struggle is to "allow students time to 'tinker' with problems" (p.1). Students need time to uncover new information, challenge inconsistencies, and struggle with ideas while engaging with problems. They do this with technology tools, manipulatives, peer discussions, connections and whole class Math Talk.

Two tools that support productive struggle are grit and a growth mindset. Grit encompasses passion and perseverance (Duckworth, 2016); a growth mindset is the belief that dedication and hard-work are essential for success (Dweck, 2007). When teachers use productive struggle through the lenses of grit and growth mindset, students in middle schools thrive.

Grit and a growth mindset enhance and empower students to reflect on their practices and acquire personal skills. Bell and Pape (2014) examined the impact social interaction has on self-regulation of students in a mathematics classroom. They examined self-regulation through using three elements as tools for problem solving: "forethought, performance, and self-reflection" (Zimmerman, 1994, 1998, 2000 cited in Bell and Pape, 2014, p. 24). Researchers examined seventh graders' use of self-regulation tools to enhance how they approached and communicated about problems. "Throughout activities and discussions, students were pressed to talk about thinking processes and about making sense of the mathematics—finding 'correct' numerical answers was de-emphasized. Students were encouraged to explain strategic thinking before providing answers to problems" (p. 28). Collaboration among students in this study enhanced their "self-regulation."

In 2015, Capraro, Capraro, Carter, and Harbaugh examined middle school students over the course of two years to identify students' growth in conceptual understanding. Teachers intentionally implemented questioning and probing techniques, as well as the use of representations in their teaching of rational numbers. Their research addressed all students, which focused on the essential attribute of being equitable about intentionally addressing students' growth. Over the course of two years, students' performance increased which was attributed to the questioning and probing techniques they were addressing.

Styron and Nyman (2008) reflected on the power and impact of the school environment on student success. They examined the school environment through three elements: community building, structures, and instructional strategies. The researchers examined over 250 surveys from teachers in nine middle schools identified as high and low performing schools. What they found was low performing schools' scores were high on middle school structures and instructional strategies and that high performing schools' scores were high on community building. Implications of their findings were: "Administrators must proactively initiate teambuilding activities to encourage healthy interactions" (p. 13). These results indicate that middle school structures are key; however, community building, inside and outside of the classroom, are essential to stable student performance. Utilizing community assets in the classroom is an effective approach to promoting grit and perseverance. Students who are struggling with a math problem can work collaboratively with a partner and often have greater success.

In each of these studies, the attributes associated with *Successful Middle Schools: This we believe* (2020) and *Principles to Action* (2014) are used. Fostering a learning atmosphere that is relevant, participatory, and motivating for all learners provides a framework for developmentally responsive, challenging, equitable, empowering and engaging work. Addressing the importance of questioning and self-regulation as tools for teaching and for student learning enhances a classroom environment that extends engagement and empowerment for students. For middle level math teachers the experiences that are presented, the level of

questioning provided, the structures that are put in place, and the opportunities to engage in communication enhance a productive classroom environment.

**Exploring Math Problems that Embrace Productive Struggle**

Teachers are often too quick to rescue students when they experience frustration or provide little guidance and coaching to help students move to the next level. They often mistake "problems with words" for those that encourage critical thinking and problem solving. Instead, teachers should look for problems that effectively utilize academic language, promote effective discourse, and engage students in deep thinking. The questions teachers ask will allow students to address challenges and will empower them to persevere.

What if teachers set up structures in their classrooms (F2F and virtual) that embrace a growth mindset, grit, and productive struggle? What if they focus on instilling passion and perseverance? What if their purpose for feedback is to infuse academic feedback, to inspire and celebrate opportunities to struggle? By using these approaches, teachers will be ready to embrace essential attributes through the use of productive struggle, grit, and a growth mindset.

For every experience, teachers should start by providing meaningful problems with multiple entry points for students. Multiple entry points refers to Howard Gardner's Entry Point Approach theory (1991), which describes the multiple paths a student can take toward knowledge and understanding. Gardner proposed that teachers should provide students no less than five approaches to solve a problem and that those approaches should vary in intelligence and modalities. Combine this with a classroom climate that values mistakes, celebrates various approaches in problem solving and teachers have the formula that engages students and provides them with experiences that allow them to think deeply and communicate richly.

The following two sample problems provide illustrations and structures that promote productive struggle, grit, and a growth mindset.

**Example 1: Sum of 15 Grid Problem**

The first example is a "game" that allows for multiple entry points and exemplifies productive struggle. Part of the "game" is that students must share their thinking on a spreadsheet so they are talking about Standards of Mathematical Practice as a tool for communicating with one another. Students can work individually or collectively share their thinking on a spreadsheet. As they engage in conversation, teachers have the opportunity to share how mathematicians often start with trial and error and then look for patterns.

***Setting the Stage (Directions)***

Show a 3 x 3 grid with the numbers one to nine placed in the cells. (See Figure 1.) Pose the problem, "Sum of 15 Grid." Using the digits 1-9, arrange them so that the sum of each row, column, and main diagonal is 15.

| | | |
|---|---|---|
| 1 | 2 | 3 |
| 4 | 5 | 6 |
| 7 | 8 | 9 |

***Figure 1. 3 x 3 Grid with Numbers 1 - 9***

***Supporting Students in the Process of Problem Solving***

Some students may need actual manipulatives. Paper tiles or index cards allow students to rearrange the digits manually. (See Figure 2a.) Virtual tiles such as https://toytheater.com/place-value-cards/ also

provide students the opportunity to manipulate the digits. (See Figure 2b.)

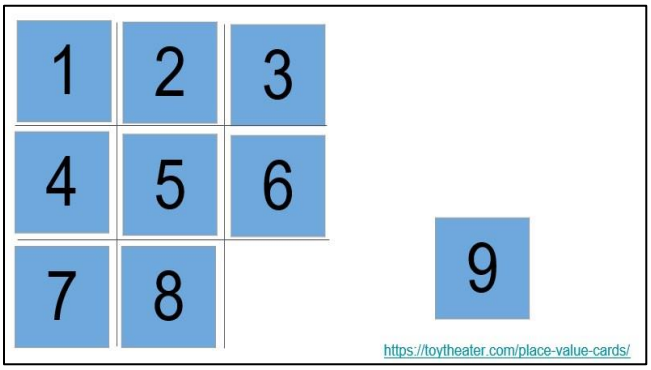


***Figure 2a. Digit Cards***

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***Figure 2b. QR Code for Virtual Digit Cards***

Engaging students in critical thinking goes beyond having them echoing back what the teacher has said. This problem promotes critical thinking in a puzzle type format. A skill of teachers is to provide structures to capture their thinking.

One structure to capture their problem solving "thinking" is to have students record their thinking each time they examine their grid. In a spreadsheet (See Figure 3.), set an area where students keep track of the process of their thinking. This form of structured communication helps students record their thinking as they work through the problem. Figure 3 demonstrates how one student documented her steps in the process of thinking through solutions (See Figure 3.) Her struggle to keep an "almost perfect" grid is revealed in the comments. This "struggle" halted her progress temporarily as she indicated mild frustration. However, she experienced some success with finding all but one sum of 15. Wrestling with this problem provided her with time and evidence of her progress.

According to Vygotsky, at the peak of frustration, students need support to persevere (Granberg, 2016). Utilizing the functionality of the spreadsheet allowed the student to keep the imperfect grid in place as a backup while she moved forward with a duplicate grid. In Figure 4, the student has new revelations regarding the 9. These revelations ultimately lead her to the final solution. As teachers, we must resist the urge to rescue students who are on the verge of a solution.

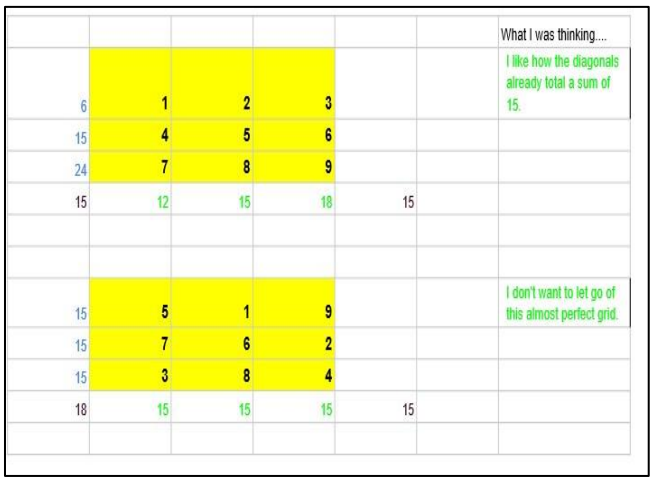


***Figure 3. Spreadsheet of Student Work with Comments***

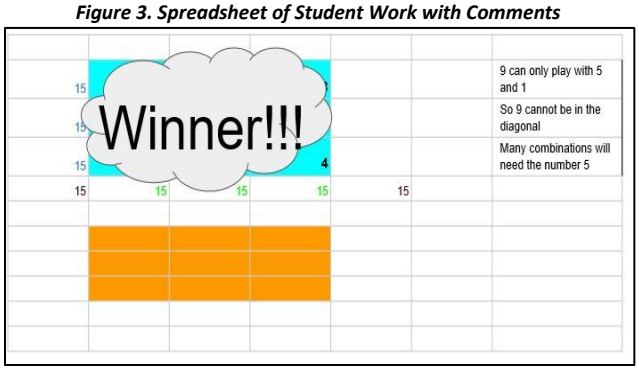


***Figure 4. Spreadsheet of Student Work with More Advanced Thinking***

Experiences that engage students in productive struggle include puzzles, opportunities to explore problems using manipulatives, and allowing students to collaborate in teams. Each of these provide different ways to engage students. The spreadsheet allowed students to communicate with one another, allowed teachers to communicate with students, and allowed the whole class opportunities to reflect and share the processes that illustrate deeper thinking. When teachers create structures for communication

with students, give students' feedback, and set the stage to share and discuss the power and impact of a growth mindset, they are providing opportunities and structures that support engaged learning.

**Example 2: Fencing an Outdoor Space**

Leveraging meaningful math tasks, mistakes, and math talk are critical avenues for learning. Posing math tasks can actively engage students, strengthens their mathematical thinking, and promote a growth mindset. Developmentally appropriate tasks give students opportunities to struggle with sufficient scaffolding and provides young adolescents with the ability to take risks beyond their imagination. Math talk as a planned support provides students space to share their thinking with one another. This strategy is critical to helping students think more deeply (Burns, 2015; 2021; NCTM, 2020, CC).

This next problem engages students in a real-life application as a way to add relevancy, a key feature of developmentally appropriate problems. The concepts in the problem relate to perimeter and area, but in a non-routine way. The example illustrates a problem with multiple entry points, involves creative thinking, and allows students to communicate their knowledge. When teachers allow time for students to engage with classmates and investigate their own thinking, they empower young adolescents to embrace their thinking and cultivate multiple strategies toward problem solving.

***Setting the Stage***

Pose the problem. (See Figure 5.)
Given a perimeter of 10 units, design a fence for a rectangle outdoor space.
What are the rectangle's dimensions?
What is the area of the outdoor space?
Draw a model to support your response.

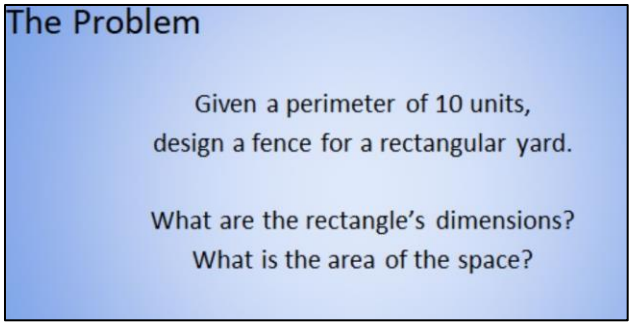


***Figure 5. Problem: Fencing an Outdoor Space***

***Supporting Students in the Process of Problem Solving***

Two structures are intentionally utilized in this example. The first is incorporating manipulatives such as geoboards, color tiles, and virtual manipulatives. These tools provide visuals to engage students in struggle and perseverance. The second is an integration of children's literature into the process of problem solving. For example, in Cindy Neuschwander's Sir Cumference book series, the heroes of the stories model productive struggle, grit, and a growth mindset while on problem solving adventures. (The example at hand describes using *The Isle of Immeter* (Neuschwander, 2006).) When teachers connect problems to thinking and provide context with children's literature, mathematics is no longer being examined as an algorithm, but as an example for becoming mathematical thinkers. These planned supports help scaffold student motivation and conceptual learning.

After the problem is posed, students begin their approach. One support is to create and analyze potential shapes on a geoboard. (See Figure 6a.) Physical geoboards and virtual options (such as the one provided by ToyTheater at https://toytheater.com/geoboard/, Figure 6b) allow students to investigate many figures in a short amount of time. Some students may be willing to persevere when given a tool to create and investigate their thinking. Students may also use models by drawing representations or designing with color tiles. Enlisting multiple approaches values mistakes and can inspire various perspectives. Varying the learning avenues also addresses the diverse needs of all students.

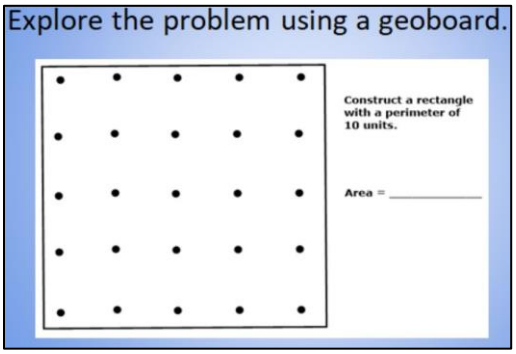


*Figure 6a. Problem Solving on a Geoboard*

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*Figure 6b. QR Code for Virtual Geoboard*

Allowing students to think without assessing the first attempt as right or wrong is a skill teachers must develop. In the example below, (Figure 7), a student responded with some thinking that needed to be refined. Teachers can use a developmentally appropriate response, in the mathematics classroom, by encouraging students to start with their own ideas and engaging them in communication. The example below does include a perimeter of 10 but the shape is not a rectangle.

In the process of assessing students' responses, teachers can identify gaps and/or misconceptions in students' thinking. Reinforcing the elements of the problem and solution by asking questions instead of "telling" answers supports productive struggle. In this case the teacher asked the student to describe the shape. Allowing students the opportunity to self-regulate by communicating their thinking allowed the student to recognize their mistake. Math talk allows time for students to reflect then revise their approaches. Teachers must listen and use communication strategies to welcome responses and celebrate thinking, correct and incorrect ideas, to inspire a growth mindset.

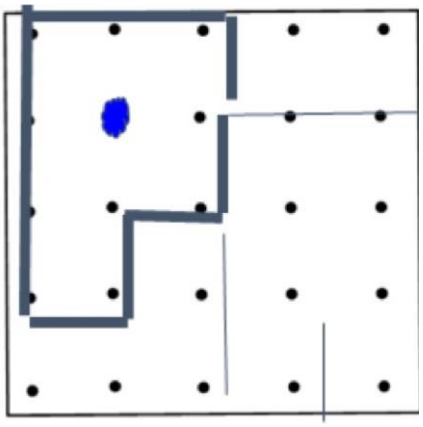

*Figure 7. Student Response*

Teachers can provide a rich problem-solving environment with support, time, and an encouraging environment that has discussion as its foundation. By observing student responses, teachers and students can celebrate mathematical thinking. Using questions to prompt feedback leads to further thinking and supports deeper thinking which leads to a stronger conceptual understanding as well as problem solving. For example, teachers can ask: "What did you learn in your 1st attempt? How did your strategy change in other attempts?" Allowing students to reflect on what they have done previously empowers them to create their own their knowledge. Math talk among peers also encourages continued thinking of multiple solutions. The impact of having students discuss different answers prompts more in-depth thinking and refinement of solutions.

Inspiring and celebrating students' opportunities to struggle by highlighting creativity in problem solving leads to a culture of mathematical dialogue. For example, in the student response above, the perimeter is 10 but the shape does not qualify as a rectangle. The student described her process of solving the problem this way: "I used a guess and check method. By counting as I went, I was able to close the figure. My figure is the shape of an elephant's head. His body is also a perimeter of 10." The student's response offers an opportunity to

discuss terminology while embracing the creativity of seeing possibilities in the design.

Part of problem solving relates to asking students if their answer makes sense (Polya, 1965). During the conversation, the student recognized her mistake and revised the elephant's head. The teacher then asked, "What other shapes could we design having a perimeter of 10 units?" This sparked further exploration. Teachers who encourage students by treating a mistake as an opportunity to learn create an open culture that empowers students to take risks in their problem solving.

With the fencing problem, there are two possible solutions that may be discovered utilizing the geoboard. To spark math talk, students can share and critique each other's designs. The teacher can ask, "Are there multiple solutions?" Students may be surprised to see multiple rectangles as their peers share. During math talk, teachers challenge students to find all possible solutions.

Under the constraints of the geoboard, dimensions 1 by 4 and 2 by 3 each has a perimeter of 10 units. (See Figure 8.) The teacher can ask, "What is the impact on the area of each rectangle? What would be the benefits of each design? What dimensions maximize the area?" Without the constraints of the geoboard, other dimensions with decimals and fractions could be introduced. In order to expand the question, the teacher can ask: "What is the greatest area you can make from a rectangle that has a perimeter of 10 units? How do you know?"

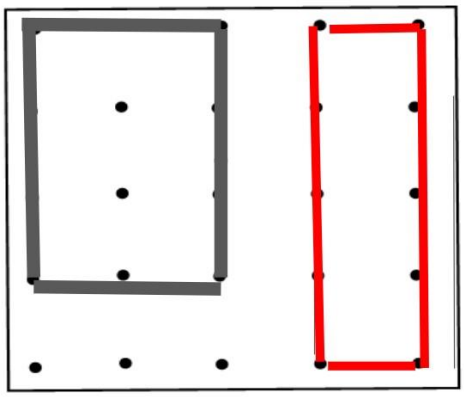

***Figure 8. Rectangles with Perimeter of 10 units***

Another entry point for productive struggle with this problem is using children's literature. Literature can provide a context for mathematical reasoning and in this context teachers could use Cindy Neuschwander's *The Isle of Immeter* (2006). In this medieval adventure, the game "Inners and Edges'' offers opportunities to discover elements of perimeter and area through a journey with the heroes of the story. As an extension, creating and playing the math game "Inners and Edges" in the classroom will actively engage students in exploring concepts in a fun way. Cross-curricular connections and games provide context for problem solving. The literature connection to the perimeter problem provides a deeper connection and understanding of the problem.

**Conclusion**

Mathematics educators who are committed to working with all of their students encourage students to persevere as a means to build confidence in their problem-solving abilities. They believe that it is important to provide intentional experiences and opportunities for all students to describe, discuss, and justify their thinking while working through math problems. They are committed to using meaningful problems that support academic language and building mathematical literacy rather than just problems with words. They believe a problem becomes meaningful when it allows for multiple pathways to a solution and challenges students to predict, explain, or describe their thinking using evidence from the problem. When teachers take the time to allow students to dig deeper, persevere, when they ask questions, allow students to work collaboratively, students' conceptual understanding expands and their critical thinking and problem solving skills improve.

Teachers who create regular opportunities to cultivate grit and productive struggle are preparing students to be future problem solvers in all areas of life. Engaging students in critical thinking goes beyond mimicking what a teacher has said; rather, effective engagement provides time and opportunities for

students to interact individually and collaboratively with content, problems, and one another.

In addition, including planned supports fosters perseverance and enhances student engagement. These structures allow for scaffolding so all students can examine, reflect, and communicate their thinking. Intentionally addressing a growth mindset, productive struggle, and grit allows teachers to collaborate and teaches students the importance of working with others as mathematical collaborators in middle schools.

Finally, perseverance is when students keep going and do not give up when things get hard. If students perceive problems as "too hard" and are not provided support, they may not continue to try or persevere. Teachers can provide planned supports that enhance communication, embrace experiences, engage in feedback to stimulate ideas, and create time for collaborative opportunities. Using Sheets, application software, manipulatives, children's literature and art are a few ways to provide responsive tools that engage, empower, and create experiences that challenge students. These tools address equity and highlight the essence of engagement. The essential attributes of successful middle schools lead teachers toward their own commitment as middle school math teachers.

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Nancy Ruppert taught middle school math for 18 years in FL, GA, SC, and NC. She is currently the chair of the education department at UNC Asheville.

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# Individual math tool bags and concrete-to-pictorial-to-virtual-to-abstract

by Glenna Tabor

**Abstract**

"Mathematics, you see, is not a spectator sport. To understand mathematics means to be able to do mathematics" (George Polya). Keeping that in mind, this 2021 edition of the Manipulative Corner about individual math tool bags and the sublevels of virtual manipulatives will be offered in an interactive, virtual format. You may read key ideas in the following paragraphs or move directly to a virtual experience by clicking on this link to the video (Tabor, 2021).


*Image 1: FREE Video Lesson, Individual Math Tool Bags and CPVA*

FREE Video Lesson link

**Math Manipulatives**

Since ancient times, people of many different civilizations have used physical objects to help them solve everyday math problems. In 300 BC, the Babylonians used counting boards. In 1837, Froebel's maneuverable gifts were introduced in kindergarten. In 1989, NCTM standards called for the use of manipulatives. In SCCTM's February, 2020 MathMate, Ryan Higgins continued to emphasize the importance of math manipulatives by introducing the first Manipulative Corner.

If you want to learn more about the history of manipulatives in math, check out this Padlet timeline.

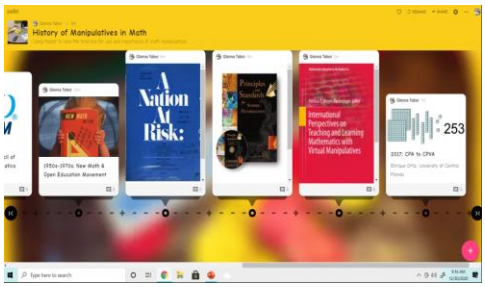


*Image 2: Padlet Timeline, History of Manipulatives in Math*

In 2021, teachers are not questioning the need for manipulatives, they are more concerned with the viable use of manipulatives in a pandemic situation. Teachers know research proves that students learn best when they begin at the concrete level, but their students can no longer use the same set of manipulatives. What about budget cuts and lack of funding?

**Math Tool Bags**

A simple solution for the manipulative challenges in 2021 is to provide each student with an Individual Math Tool Bag. The Individual Math Tool Bag, described in the video lesson (Tabor, 2021), gives students the concrete tools they need to explore abstract concepts. Since the bag is designed to come

and go in their backpack, students can use manipulatives no matter where they are sitting.

With budget cuts and lack of funding, the contents of these Individual Math Tool Bags are less expensive household items that are easier to obtain. Download the Contents of an Individual Math Tool Bag and Suggested Activities for Math Tools from my blog on glennatabor.com. Investigate a sampling of the use of these household manipulatives, at all levels, in the video lesson (Tabor, 2021).

Blog link


*Image 3: Blog Post on GlennaTabor.com*

**Virtual Manipulatives**

In a 2002 article for Teaching Children Mathematics, Moyer, Bolyard and Spikell answered the question, what is a virtual manipulative? In 2016, the definition of a virtual manipulative was further refined as, "an interactive, technology-enabled visual representation of a dynamic mathematical object, including all of the programmable features that allow it to be manipulated, that presents opportunities for constructing mathematical knowledge" (Moyer-Packenham & Bolyard, 2016)

In 2017, Enrique Ortiz conducted a study that further delineated the virtual level of math manipulatives into three digital-dynamic sublevels: virtual-Concrete, virtual-Pictorial and virtual-Abstract. To be considered a virtual manipulative, it must be dynamic. Static virtual models do not move and are not considered "true" virtual manipulatives (Duebel, 2010).

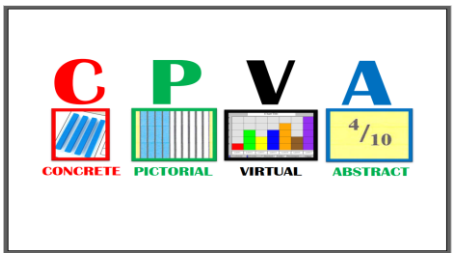


*Image 4: CPVA, Concrete-to-Pictorial-to-Virtual-to-Abstract*

The refinement of Virtual into three digital-dynamic sublevels is quite powerful in sophisticated remote instruction, especially when only virtual tools are available. You can challenge yourself by identifying the correct sublevel of virtual manipulative in the virtual lesson examples.

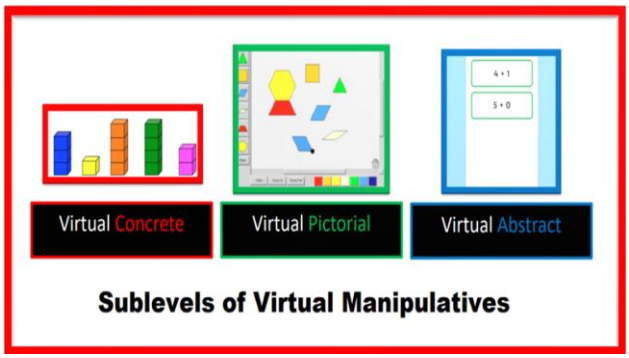


*Image 5: Sublevels of Virtual Manipulatives*

**Why?**

Learners throughout time have benefited from using manipulatives as they explored mathematical concepts. The need for manipulatives has not changed because of social distancing or remote instruction. A teacher who includes the use of dynamic, digital manipulatives and individual math tool bags is giving their students a gift of understanding, exploration, engagement, and, most of all FUN!

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**About the Author**

Glenna Tabor's lifelong mission has been to motivate, educate, and transform the lives of everyone she meets – even in a virtual setting! She has been fascinated with math manipulatives, at all levels, since holding her first one when she was in college. She'd love to hear what new tools you add to your students' Individual Math Tool Bags and how you incorporate virtual math manipulatives into your math instruction. glenna@glennatabor.com.

# Math Trail: Heritage Shores Nature Preserve, North Myrtle Beach, South Carolina

by Cindy Parker


Just a short drive from Main Street in North Myrtle Beach, a few miles away in Cherry Grove, is a hidden gem, Heritage Shores Nature Preserve. One of the most unique parks in the city in North Myrtle Beach, Heritage Shores Nature Preserve was created in 2007 with seven (7) acres of walking paths. The park is located on an island that extends into the Cherry Grove Marsh. This land has been preserved in its native state and is accessible via a series of elevated boardwalks and observation docks. Visitors will find interpretive signage throughout the park describing the plants, animals, fish, and birds that can be seen within the park. Park amenities include the following:

- 40 stations along an interpretive trail with native soil primitive walking paths
- Raised wooden walking path through the marsh areas of the preserve
- 2 picnic/shade shelters
- Stationary dock providing viewing access

**Directions to Heritage Shores Nature Preserve**

Address: 5611 Heritage Drive, North Myrtle Beach, South Carolina 29582

Directions: Take Ocean Boulevard and turn onto 53rd Avenue North, toward the boat landing, away from the Atlantic Ocean. Upon crossing the narrow one vehicle at a time, House Creek Bridge, 53rd Avenue leads directly to the Cherry Boat Ramp. The brick lot parking for the Heritage Shores Nature Preserve is located at the western end of 53rd Avenue North, along the creek, to the right of the boat landing area. After parking, it's a four-block walk along the sidewalk, away from the boat landing area to the entrance of the Preserve.

**The Heritage Shores Preserve Math Trail**

A **math trail** is a walk with various stops where the participants look at **mathematics** in the world around them, as they ask and answer questions about their discoveries. **Math trails** are a wonderful way to stimulate interest in **mathematics** and to explore the outdoors at the same time.

No tools are needed for a math trail, other than perhaps some string, a tape measure, a notepad, and a pencil, so children are free to roam, observe, consider, reason, and internalize a new appreciation of the math embedded in their natural environment. While on a math trail, students develop mathematical and environmental literacy while having fun. A math trail enables children to engage with mathematical experiences in the real world and to gain first-hand knowledge of how math can be used to interpret the world in which they live, while illuminating new mathematical concepts that complement and extend what they have learned in textbooks and within a classroom.

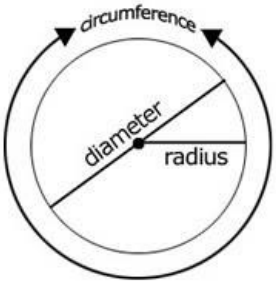
**The Heritage Shores Walking Trail**

To begin the Math Train adventure, bear to the right when the trail branches left and right. Our sea turtle friend is headed right, so be sure that you go in that direction as well.

**Mathematical Activities for The Heritage Shores Math Trail**

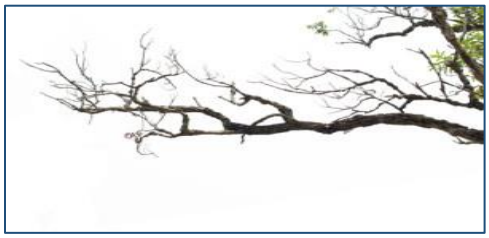
1. Your first mathematical activity is to keep a tally of interesting things that you think that you "*might see*," during your adventure. At the end of the trail, you should create your choice of a bar graph, pictograph, circle graph, or other appropriate statistical graph or chart using your data.
2. When you arrive at the first wooden walkway along the trail, stop before you cross it. Estimate how many steps you think that it will take you to get across along the walkway. Write your estimate down. Now, walk naturally and count your steps as you walk across and write the actual number of steps that you took. Compare your estimate to your number of steps. How well did you estimate? Why do you think your estimate and the actual number of steps turned out as they did?
3. Symmetry is when two halves of something are exactly the same. A butterfly has symmetry, as does a heart. As you walk along the trail, notice the symmetry that you see in the leaves and plants. Draw some of the items that you see that have symmetry and sketch in the "line of symmetry" in your drawings.

4. As you walk along the trail, look for fallen logs or posts along the trail. You should now explore the relationship (ratio) of the circumference (the measure around the tree or post) to the diameter (the distance across the tree or post at its very center). Take any item you have that could be used as a measure (a string, a belt, a towel, or a scarf). Measure across the tree's diameter and keep that measure marked with your finger or by tying a knot in the scarf or string. Using that marked measure, measure how many times you can go around the circumference of the tree or post. Repeat this with three or four trees or posts of different diameters. What have you discovered about the measure of the circumference to the diameter? circumference: diameter




5. As you walk along, begin to take notice of tree branches. Leonardo Fibonacci discovered the pattern in about the year 1200. Each number in the sequence comes by adding the previous two numbers. **0, 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987 …** For instance, the number 13 is achieved by adding the numbers 5 and 8 and the number 21 is achieved by adding 8 with 13.

The Fibonacci Sequence is actually a numerical representation of the fabled 'golden ratio' which can be found in art and nature, comprising everything from flowers to seashells and even the hairs on your head. When charted on a graph, that sequence spirals outwards, like the branches of a tree.



- The Fibonacci sequence is so widespread in nature that it can also be seen in the way tree branches form and split.
- The main trunk of a tree will grow until it produces a branch, which creates two growth points. One of the new stems will then branch into two, while the other lies dormant. This branching pattern repeats for each of the new stems.

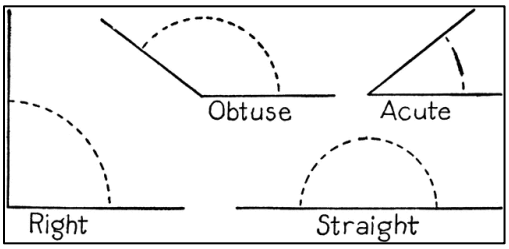
Look for and list other items in nature that you find here in Heritage Shore Nature Preserve that are examples of this mathematics in nature miracle, the Fibonacci Sequence. You'll see some more examples below to help give you some ideas.

This miracle of math and nature has been used throughout history in many works of art such as the Mona Lisa, but it doesn't stop there, the Fibonacci sequence can even be heard in music. An interesting music example is Lateralus by the heavy metal band Tool.

"Lateralus's title-track's introduction lasts one minute and 12 seconds and the numbers 0, 1, 1, 2 are the first four in the Fibonacci sequence. The first verse kicks in on 97 seconds, which is approximately 1.618 minutes, i.e. the golden ratio. Each verse is also 55 seconds long, which is the 11$^{th}$ number in the sequence. The syllables in those verses match the sequence too, peaking at 13 (coincidentally, the same number of total album tracks) and then again in a descending order thereafter. Furthermore, the time signature of the

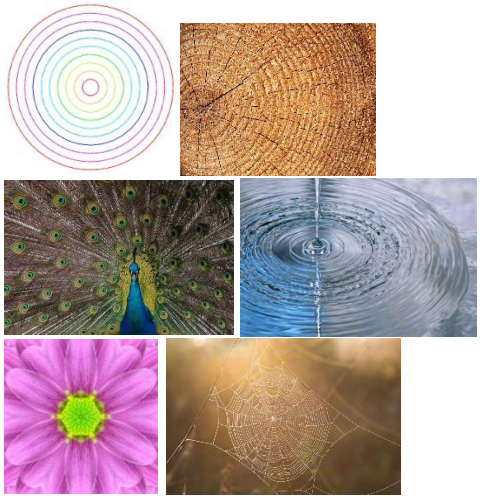
song's main riff is 9/8, 8/8 and 7/8, and 987 is the 17th number in the sequence. The final lines Maynard sings are, '*Spiral out. Keep going*.'." (https://en.wikipedia.org/wiki/Lateralus_(song)

6. When you are about halfway around the island, you will see a shelter with a picnic table along the edge of the water of House Creek. Look for the geometry that was used in the building of this shelter and table. Write down the geometric shapes that you can see. Look for the angles that were in the construction.

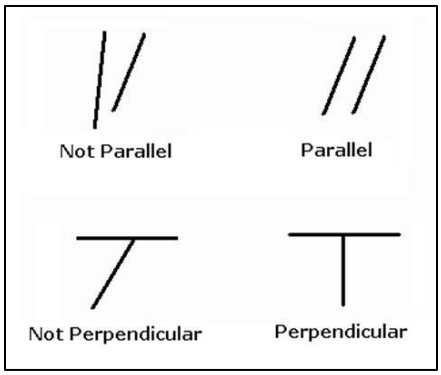


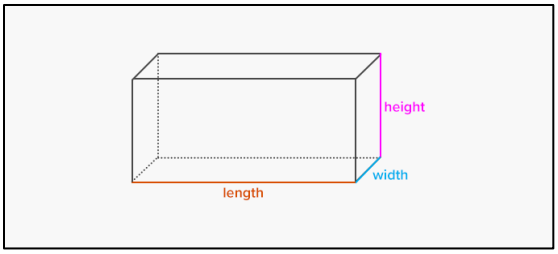

How many of each type of angle do you see? How could you measure some of the angles that you have found?

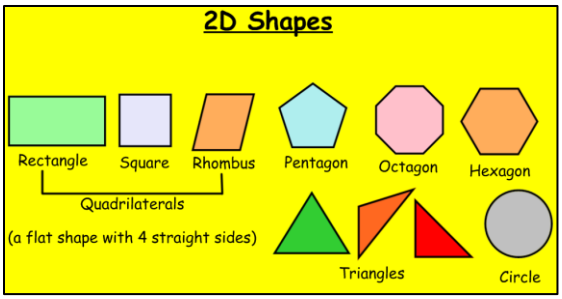
7. After leaving the picnic shelter on the point of the island, continue along the trail. Look for more picnic tables. How many picnic tables are there on the island? How many people could sit at each table? How many people could have a picnic lunch, sitting at a table on the island at the same time?
8. A common shape in nature is a set of concentric circles. Concentric means the circles all share the same center, but have different radii. This means the circles are all different sizes, one inside the other. A common example is when we see the ripples in a lake or pond when something hits the surface of the water. We also see concentric circles in the layers of an onion and the rings of trees that form as it grows and ages. Orb spider webs also have concentric circles. Write down the concentric circle that you have seen along the trail.



9. Did you know that a tree's age can be estimated by measuring the trunk? Measure the circumference of a tree. On average, the circumference of a tree grows at about 1/2 to 3/4 inches per year. How old is the tree that you measured?
10. Docks, bridges, and buildings have several geometric details that can be described, including parallel and perpendicular lines, angles, and measurement of their dimensions, as wells as their shapes. In how many ways can you describe the docks on this island mathematically?






The North Myrtle Beach Chamber & CVB and the South Carolina Council of Teachers of Mathematics hope that you have enjoyed your visit to the Heritage Shores Nature Preserve in North Myrtle Beach, South Carolina. By the time that you have completed your mathematical walk, you'll have noticed that you arrived back to where your walk began. If you have any other ideas that could be added to this mathematical adventure or if you would like to help develop and design other mathematical trails or science trails, please contact Cindy Parker, M.Ed., Executive Director of the South Carolina Council of Teachers of Mathematics by emailing: director@scctmconference.org

**About the Author**

A native of South Carolina, Cindy Parker taught mathematics and later engineering and robotics at Alice Drive Middle School in Sumter and worked as an adjunct instructor with the College of Charleston and the University of South Carolina teaching technology and math methods classes. In the past, she has volunteered with SCCTM by presenting at a number of state and national conferences, worked as Middle School Vice President, President-Elect, Program Chair, and Registrar. Retired, she now lives in North Myrtle Beach where she volunteers with the North Myrtle Beach Sea Turtle Patrol and serves the South Carolina Council of Teachers of Mathematics as its Executive Director.

# Proportionality is hidden everywhere. Can you find it?

by Carrie Simpson

In 7th grade math, students are asked to identify and model proportional relationships through a variety of representations. This lesson is intended to be an introductory/investigative lesson to ratio and proportional relationships. For students to understand why they are learning the content presented, they need to see how it applies to the world around them. Using the children's book, *If you Hopped Like A Frog*, proportional relationships are presented with things students can visualize and relate to throughout the story. Upon completion of the story, students are asked to research four real world examples where they will compare two quantities that are proportional (like those they read in the story). The information they gather will be placed in a presentation platform. The rubric created is intended as a checklist for teachers to use to ensure that students understand the concept taught before moving forward to more complex proportional problems.

## Lesson Plan

**Standard**

7.RP.2. Identify and model proportional relationships given multiple representations, including tables, graphs, equations, diagrams, verbal descriptions, and real-world situations.

a. Determine when two quantities are in a proportional relationship.

**Objective**

Determine when two quantities are in a proportional relationship.

**Essential Question**

How can you determine when two quantities are proportional?

**Lesson Outline**

1. The teacher will read aloud the children's book "If you hopped like a frog" written by David M. Schwartz and illustrated by James Warhola.
2. Upon completion of the book, have a whole group discussion about the proportional relationships found within the story.
3. Continue the lesson by providing sample mathematical problems involving quantities that are proportional and show how they are proportional.
4. Provide an additional example such as: The approximate length of the titanic is proportional to the approximate length of three football fields (comparing length to length).
5. Inform students that they are to research four real world examples where they will compare two quantities that are proportional (similar to those read in the story). Their information gathered will be placed in a presentation platform. In their presentation, each of the 5 quantities must include the following:
    a. A verbal description of the examples providing the mathematical measurements
    b. A picture of examples showing visually how the items are proportional
    c. Math proving that the quantities are proportional

6. Allow students to work with a partner on a project involving real-world examples of proportional relationships. Give them the following ways to choose how they would like their information to be presented:
    a. Virtual presentation (google slides, prezi, powtoon, etc.)
    b. Poster board
    c. Children's book
    d. Pamphlet or informational brochure
7. Once all groups have completed the project, have them present their information to the class.
8. Use a 4-point rubric to grade students (see suggested rubric below)

| **4**<br>**Exemplary** | **3**<br>**Met** | **2**<br>**Approaching Proficient** | **1**<br>**Needs Improvement** |
|---|---|---|---|
| The student has mastered the targeted objective.<br><br>The student is able to determine when two quantities are in a proportional relationship without error. | The student has met the targeted objective.<br><br>The student is able to determine when two quantities are in a proportional relationship with minimal error. | The student is approaching the level of the objective with guidance.<br><br>The student is able to determine when two quantities are in a proportional relationship when prompted and provided guidance. | The student has not met the learning objective.<br><br>The student is able to provide little or no evidence of determining when two quantities are in a proportional relationship. |

**About the Author**

Carrie is an alumni from Lander University and Anderson University and holds a Bachelor's Degree in Elementary Education and a Master's Degree in Administration and Supervision. She is Nationally Board Certified in Adolescence Mathematics. Carrie currently teaches 7th grade math at Robert Anderson Middle School and serves as the 2020-2021 District Teacher of the Year for Anderson School District Five.

# Using Children's Literature to Teach Middle Grades Mathematics

by Candice Brucke

Sometimes making decisions in life is complicated. Sometimes it is comfortable. There are often many factors to consider before settling on a final solution when it comes to making choices. Many people check consumer reports before buying a new car or a new refrigerator. Some people ask friends and family for their opinions before deciding to try a new restaurant. Others just go into a situation blindly and make decisions based on a "feeling" or experience. In the end, a person has to determine if squeezing the orange is worth the juice.

As mathematics teachers compete for students' attention in an ever-evolving technological world, they have to ask themselves if incorporating pieces of children's literature into their middle grades math classes is worth the time and effort. Math teachers at all levels of public education can use children's literature to introduce, reinforce, or close out a mathematical lesson, activity, or unit of study. If you have been teaching for any amount of time, you are aware that mathematics is its own language. Incorporating children's literature can make communicating in, with, and by mathematics easier, more comprehensible, and more fun. Educational research supports using literature to teach mathematics. By using the strategy of problem-posing with quality math-related literature, teachers can encourage a wide range of mathematical communication in their classrooms (Whitin & Whitin, 2004).

Not only does adding children's literature promote mathematical communication, but finding the right piece, be it a poem, a picture book, or a novel, can foster multicultural awareness. Many of the Newberry award-winning books are ethnically and culturally rich (Moen, 1994). Adolescent literature can be used to motivate students, provoke interest, connect mathematical ideas, promote critical thinking skills, and provide a context that leads to problem-solving (Welchman-Tischler, 1992).

In adolescent literature, the stories and vocabulary are recognizable, easy to comprehend, and are on students' independent reading levels (Osborn, 2001). The familiar storylines, predictable outcomes, and informal language can allow students to enjoy the math concepts and activities that go along with the stories. Savvy teachers can use reputable pieces of math related literature to illustrate the South Carolina College and Career Ready Standards for Mathematics.

It is important to connect the literature to mathematics, either by making connections to the characters, the storyline, or by incorporating the literature elements into the assignment. "Without a meaningful context, learners may come to regard mathematics as an abstract and irrelevant system" (Whitin & Wilde, 1992). A teacher can choose literature pieces, like short books, poems, plays, and newspaper articles that take 10-15 minutes to read aloud. Anything shorter may not allow a teacher to fully develop a math concept or introduce an idea. Anything longer makes a teacher compete with short attention spans, lose the "punch" of the literature piece, and forfeit valuable time that could be used for introducing, teaching, and/or practicing the math concept or skill.

Literacy instruction must be an integral part of a middle grades math class so that mathematics is not taught in isolation. By being able to effectively communicate in mathematics, students can transfer this skill into other disciplines, such as language arts, science, and social studies. Picture books can provide springboards to discussions and are great for sparking ideas for student writing opportunities, especially in math class.

As a former middle grades mathematics teacher, I chose to use adolescent books because of their length and familiar storylines. The pieces of literature I used are written on students' independent reading levels and they tend to trigger appropriate mathematical discussions. Some of the lessons I created served as a Launchpad for creative writing opportunities as well.

Before embarking on a journey to incorporate children's literature into a middle school mathematics lesson, I encourage you to consider books that you are familiar with and are interested in first. Enthusiasm is contagious and students can discern authenticity. Do not try to fake that you like the idea of using the traditional story of Cinderella to introduce the concept of slope if you really do not think it will work. "Your own enthusiasm for titles you select can inspire the same in your students" (Moen, 1994).

Following this article is an Understading by Design lesson plan formatted to reflect the research of Wiggins (1998). I coupled *The Keeping Quilt* (Polacco, 1988) with proportional reasoning math problems. This lesson was designed to be used to close out a mini-unit on proportional reasoning. Go ahead, squeeze the orange. It's worth the juice.

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**About the Author**


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**Understanding by Design: Proportional Reasoning Closing Activity**

***The Keeping Quilt* by Patricia Polacco**

**Stage 1**

A. Enduring Understanding: Proportional reasoning is a tool used to solve real-world problems by showing relationships and making comparisons in quantitative problems.

B. Essential Questions:
   1. Why does proportional reasoning increase my understanding of the real world?
   2. How can proportional reasoning help me find unknowns in everyday life?
   3. How can properties of similarity help me solve measurement problems?
   4. When quantities have different measurements, how can they be compared?

C. Knowledge and Skills:
   1. Analyze proportional relationships and use them to solve problems.
   2. Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.
   3. Choose an appropriate strategy to solve a problem.

**Stage 2**

A. Performance Tasks: Use operations to solve equations related to the story.
B. Formal Assessments: Texts/quizzes
C. Unprompted Evidence: Teacher questioning, slate checks
D. Student Self-Assessment: Student solutions to the problems

**Stage 3**

W: Proportional reasoning is a tool used to solve real world problems by showing relationships and making comparisons in the quantitative problems
H: The class will read *The Keeping Quilt* (Polacco, 1988) and solve related problems
E: Key vocabulary: ratio, huppa, babushka, challah, kulich, equal parts, rate, weighted average
R: Reflection sheet after each assignment and assessment
E: Evaluate student problems

***The Keeping Quilt* by Patricia Polacco**

**Lesson Plan**

**Objective:**
Students will solve a variety of problems by understanding the problem, choosing an appropriate strategy to solve the problem, determine the reasonableness of the solution, and explain the solution to a classmate.

**Suggested Materials:**
*The Keeping Quilt* by Patricia Polacco
Scratch paper/calculator
Pencil

**Focus:**
Read the story to the class. Students will solve problems related to the characters in the book. Discuss the significance and relevance of the artwork in the book. Discuss family traditions, personal triumphs, and perhaps even disappointments.

**Lesson Presentation:**
Review key vocabulary: ratio, huppa, babushka, challah, kulich, equal parts, rate, and weighted average

**Model:**
The teacher will model how to solve problems with proportional reasoning using grade-appropriate vocabulary.

**Guided Practice:**
The whole class will work together to solve sample proportional reasoning problems.

**Independent Practice:**
Students will work to complete similar problems to those presented in the final assessment. Students should solve a variety of problems, choose appropriate strategies, determine each solution's reasonableness, and explain the solution to a classmate (written or verbal).

**Assessment:**
Students will solve problems using all four operations. Students should show their work and provide an explanation (in complete sentences) for each solution.

***The Keeping Quilt* by Patricia Pollacco**

**Middle Grades Math**
**Proportional Reasoning (Closing Problems)**

1. Great-Grandma Anna's family recipe for chicken soup makes 4 servings of 310 calories each. Great-Grandma Anna decided to make 1 ½ times the amount in the recipe. How many calories are in Great-Grandma's batch of chicken noodle soup?

2. The ratio of girls to boys in Great-Grandma Anna's school was 3:2. There are 134 boys in the 7$^{th}$ grade. What is the total number of students in 7$^{th}$ grade at Great-Grandma Anna's school?

3. Before Grandma Carle move to her farm in Michigan, she went to the store to buy some yogurt covered raisins. The raisins sold for $3.99 per pound. How much did Grandma Carle spend on 33 1/3 pounds of yogurt-covered raisins?

4. Grandma Carle has a car that averages 20 miles per gallon in city driving and 30 miles per gallon in highway driving. At these rates, how many gallons of gasoline will Grandma Carle's car use on a 300 mile trip if 4/5 of the distance is highway driving and the rest is city driving?

5. Mary Ellen notices her brother wore green on 9 of the last 21 days. Assuming the same rate, on how many days would Mary Ellen expect him to wear green in the next five weeks?

6. On the farm in Michigan, 2 groomers working together can brush 8 horses in 3 hours. How many hours would it take 3 groomers to brush 12 horses at this rate?

7. During a three hour quilting part, the word *huppa* was used, on average, once every five minutes during the first two hours. If the word *huppa* was used 54 times throughout the quilting party, then what was the average number of minutes between uses in the third hour?

8. A popular brand of Russian coffee costs $20 per pound, and a particular brand of Starbucks coffee costs $16 per pound. If you mix 15 pounds of Russian coffee with 5 pounds of Starbucks coffee, how many dollars does one pound of the mixture cost?

9. For every two used paperback books Patricia buys at the regular price, she gets a third book for a nickel. If Patricia spent $4.65 for nine paperback books, what is the regular price of a used paperback book, in cents?

10. On the family farm in Michigan, an average bushel of corn contains 72,800 kernels and weighs 56 pounds. There are 16 ounces per pound, and average ear of corn contains 650 kernels. In ounces, how much do the kernels from one average ear of corn weigh?

**Answer Key**

1. **1,860 calories.** Family recipe makes 4 servings. 4 X 1.5= 6.6 x 301 = **1,860 calories**.

2. **335 students**. The ratio is 3:2. Three "equal" pats of the 7th grade are girls and 2 "equal" parts are boys. 134/2 = 67. All five parts are 5 x 67 = **335 students**.

3. **$133**. The number of pounds times the price per pound will give the cost. 33 1/3 x 3.99 = $133.

4. **11 gallons**. The 300 mile trip is 300 x 4/5 = 240 miles of highway driving. 300 – 240 = 60 miles of city driving. The car uses 240/30 = 8 gallons of gasoline on the highway and 60/20 = 3 gallons of gas in the city. 8 + 3 = **11 gallons** of gasoline.

5. **15 days**. Nine days of wearing green out of 21 days is the same rate as 3 days of wearing green per week. At this rate, Mary Ellen can expect her brother to wear green 3 x 5 = **15 days** in the next five weeks.

6. **3 hours**. If working together, 2 groomers can brush 8 horses in 3 hours, then each groomer working alone can brush 4 horses in 3 hours (8/2). Therefore, 3 groomers can brush (3 x 4) 12 horses in the same **3 hours**.

7. **2 minutes**. The rate once every five minutes is the same as 12 times per hour. The word *huppa* must have been used 24 times in the first two hours and 54 – 24 = 30 times in the third hour. For the 3rd hour, there were, on average, 60/30 = **2 minutes** between each use of the word *huppa*.

8. **$19**. This is a "weighted average" problem. The total cost of the mix is (15 x 20) + (5 x 16) = 300 + 80 = $380. Since the total weight is 15 + 5 = 20 pounds, the price per pound is 380/20 = **$19**.

9. **$0.75 or 75 cents**. Nine books are three sets of three books. Patricia got three of the books for a nickel each ($0.05 x 3 = $0.15). For the other 6 books, Patricia paid a total of $4.65 - $0.15 = $4.50. When we divide by 6, we find the regular price of a used paperback book is $4.50/6 = **$0.75 or 75 cents**.

10. **8 ounces**. On average, 1 pound of corn must be about 73,800/56 = 1,300 kernels. Since an average ear of corn contains 650 kernels, the kernels from one average ear of corn must weigh 650/1,300 = ½ pound or 16 x ½ = **8 ounces**.